

Validity of Quasi-Degenerate Neutrino Mass Models and their Predictions on Baryogenesis

Ng.K.Francis^{a,b1} and N. Nimai Singh^{b2}

^a*Department of Physics, Tezpur University, Tezpur-784028, India*

^b*Department of Physics, Gauhati University, Guwahati-781014, India*

Abstract

Quasi-degenerate neutrino mass models (QDN) which can explain the current data on neutrino masses and mixings, are studied. In the first part, we study the effect of CP-phases on QDN mass matrix (m_{LL}) obeying $\mu - \tau$ symmetry in normal hierarchical (QD-NH) and inverted hierarchical (QD-IH) patterns. The numerical predictions are consistent with observed data on (i) solar mixing angle (θ_{12}) which lies below tri-bimaximal (TBM) value, (ii) absolute neutrino masses consistent with $0\nu\beta\beta$ decay mass parameter (m_{ee}) and (iii) cosmological upper bound $\sum_i^3 m_i$. m_{LL} is parameterized using only two unknown parameters (ϵ, η) within $\mu - \tau$ symmetry. The second part deals with the estimation of observed baryon asymmetry of the universe (BAU) where we consider the Majorana CP violating phases (α, β) and the Dirac neutrino mass matrix (m_{LR}). m_{LR} is taken as either the charged lepton or the up quark mass matrix. α, β is derived from the heavy right-handed Majorana mass matrix M_{RR} . M_{RR} is generated from m_{LL} and m_{LR} through inversion of Type-I seesaw formula. The predictions for BAU are nearly consistent with observations for flavoured thermal leptogenesis scenario for Type-IA in both QD-NH and QD-IH models. We also observe some enhancement effects in flavour leptogenesis compared to non-flavour leptogenesis by a magnitude of order one. In non-thermal leptogenesis QD-NH Type-IA is the only model consistent with observed data on baryon asymmetry. QD-NH model appears to be more favourable than those of QD-IH. The predicted inflaton mass needed to produce the BAU is found to be $M_\phi \sim 10^{10}$ GeV corresponding to the reheating temperature $T_R = 10^6$ GeV. The present analysis shows that the three absolute neutrino masses may exhibit quasi-degenerate pattern in nature.

Keywords: QDN models, absolute neutrino masses, leptogenesis.

PACS Numbers: 14.60.Pq; 12.15. Ff; 13.40.Em.

¹*E-mail: ngkf2010@gmail.com*

²*E-mail: nimai03@yahoo.com*

1 Introduction

The presently available tightest cosmological upper bound of the sum of three absolute neutrino masses, has come down to the lowest value, $\sum_i m_i \leq 0.28$ eV [1], and a larger value of neutrino mass $m_i \geq 0.1$ eV in Quasi-Degenerate Neutrino (QDN) mass models, has therefore to be ruled out. Furthermore, the upper bound value of neutrino mass parameter $|m_{ee}| \leq 0.27$ eV appeared in the neutrinoless double beta decay ($0\nu\beta\beta$) experiments [2], also disfavors larger values of neutrino mass eigenvalues with same CP-parity. Investigations on QDN models in normal hierarchical (QD-NH) and inverted hierarchical (QD-IH) patterns of the three absolute neutrino masses, require a detailed numerical analysis to check whether such QDN models can really accommodate lower values of absolute neutrino masses $|m_i| \leq 0.09$ eV which are consistent with the above cosmological bound [1]. In the next step, the QDN models are again applied for the prediction of baryon asymmetry (η_B) of the universe [3]. In order to estimate the observed baryon asymmetry $\eta_B = (6.5^{+0.4}_{-0.5})10^{-10}$ [3] from a given neutrino mass model, one usually starts with a suitable texture of light Majorana neutrino mass matrix (m_{LL}) and then relates it with the heavy Majorana neutrinos matrix (M_{RR}) and the Dirac neutrino mass matrix (m_{LR}) through the inversion of Type-I seesaw mechanism in an elegant way. Since the structure of Yukawa matrix for Dirac neutrino is not known, we consider the texture of Dirac neutrino mass matrix m_{LR} as either the charged lepton mass matrix or up quark mass matrix, as allowed by SO(10) GUT models for phenomenological analysis.

In many of the theoretical calculations on leptogenesis, the complex CP violating phases are usually derived from the Majorana phases appearing in PMNS leptonic mixing matrix U_{PMNS} which diagonalizes m_{LL} . Hence in such approach m_{LL} is no longer taken as the starting point. However, in the present work, we consider a different route for the origin of complex CP violating phases which are derived from M_{RR} in the estimation of baryon asymmetry of the universe. This theoretical possibility is the main part of the present investigations.

We first introduce a general classification of QDN models based on their CP-parity patterns in the three absolute neutrino masses, and then parameterize the mass matrices using only two unknown parameters (ϵ, η), which can reproduce correct predictions on neutrino oscillation mass parameters and mixing angles, consistent with the latest observational data. Though such parameterization is intuitive, it is quite realistic for phenomenological

analysis. The estimations of baryon asymmetry of the universe in the light of thermal and non-thermal leptogenesis, may thus serve as an additional criteria to discriminate the correct pattern of neutrino mass models and also to shed light on the structure of unknown Dirac neutrino mass matrix.

The paper is organised as follows. In section 2 we parameterize the neutrino mass matrix. In section 3, numerical analysis and predictions on baryon asymmetry are outlined. Finally in section 4 we conclude with a summary. In the Appendix we briefly summarize the formalism for estimating the lepton asymmetry in thermal leptogenesis through out-of-equilibrium decay of the heavy right-handed Majorana neutrinos. This is followed by flavoured thermal leptogenesis and non-thermal leptogenesis.

2 Parameterization and computations

2.1 Parameterizations of neutrino mass matrix

A general μ - τ symmetric neutrino mass matrix [4,5] with its four unknown independent matrix elements, requires at least four independent equations for realistic numerical solution,

$$m_{LL} = \begin{pmatrix} m_{11} & m_{12} & m_{12} \\ m_{12} & m_{22} & m_{23} \\ m_{12} & m_{23} & m_{22} \end{pmatrix}. \quad (1)$$

The three mass eigenvalues m_i and solar mixing angle θ_{12} , are given by

$$m_1 = m_{11} - \sqrt{2} \tan \theta_{12} m_{12},$$

$$m_2 = m_{11} + \sqrt{2} \cot \theta_{12} m_{12},$$

$$m_3 = m_{22} - m_{23}.$$

$$\tan 2\theta_{12} = \frac{2\sqrt{2}m_{12}}{m_{11} - m_{22} - m_{23}}. \quad (2)$$

The observed mass-squared differences are calculated as

$$\Delta m_{12}^2 = m_2^2 - m_1^2 > 0, \quad \Delta m_{32}^2 = |m_3^2 - m_2^2|. \quad (3)$$

In the basis where charged lepton mass matrix is diagonal, we have the

leptonic mixing matrix, $U_{PMNS} = U$, where

$$U_{PMNS} = \begin{pmatrix} \cos \theta_{12} & -\sin \theta_{12} & 0 \\ \frac{\sin \theta_{12}}{\sqrt{2}} & \frac{\cos \theta_{12}}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{\sin \theta_{12}}{\sqrt{2}} & \frac{\cos \theta_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \quad (4)$$

The neutrino mass parameter m_{ee} in $0\nu\beta\beta$ decay and the sum of the absolute neutrino masses in WMAP cosmological bound $\sum_i m_i$, are given respectively by,

$$m_{ee} = |m_1 U_{e1}^2 + m_2 U_{e2}^2 + m_3 U_{e3}^2|, m_{cosmos} = m_1 + m_2 + m_3. \quad (5)$$

A general classification for three-fold quasi-degenerate neutrino mass models [5] with respect to Majorana CP-phases in their three mass eigenvalues, is adopted here. Diagonalization of left-handed Majorana neutrino mass matrix m_{LL} in eq.(1) is given by $m_{LL} = U D U^T$, where U is the diagonalising matrix in eq.(4) and $\text{Diag} = D(m_1, m_2 e^{i\alpha}, m_3 e^{i\beta})$ is the diagonal matrix with two unknown Majorana phases (α, β) . In the basis where charged lepton mass matrix is diagonal, the leptonic mixing matrix is given by $U = U_{PMNS}$ [6]. We then adopt the following classification according to their CP-parity patterns in the mass eigenvalues m_i namely Type IA: $(++)$ for $D = \text{Diag}(m_1, -m_2, m_3)$; Type IB: $(+++)$ for $D = \text{Diag}(m_1, m_2, m_3)$ and Type-IC: for $(++-)$ for $D = \text{Diag}(m_1, m_2, -m_3)$ respectively. We now introduce the following parameterization for μ - τ symmetric neutrino mass matrices m_{LL} which satisfy the above classifications [4,5] and present a detailed numerical analysis.

2.2 Numerical analysis and predictions

For detailed numerical analysis we first choose the light Majorana neutrino mass matrix m_{LL} presented in section 2.1. These mass matrices which obey $\mu - \tau$ symmetry [4], have the ability to deviate the solar mixing angle from that of TBM [7]. Next we estimate the numerical values of the three absolute neutrino masses. As discussed before, we need to introduce the neutrino mass scale m_3 as input parameter in addition to the observed data [8] on solar and atmospheric neutrino mass-squared differences (Δm_{21}^2 and $|\Delta m_{32}^2|$). The three input parameters are fixed at the stage of predictions of neutrino mass parameters and mixing angles given in Tables 2 and 3. These results are consistent with the recent data on neutrino oscillation parameters [8]. For numerical analysis we use the best-fit values of the global

input	calculated	QD-NH		QD-IH	
m_3	ρ	m_1	m_2	m_1	m_2
0.40	0.015	0.39689	0.39699	0.40289	0.40299
0.10	0.24	0.08674	0.08718	0.11104	0.11136
0.08	0.375	0.06264	0.06326	0.09340	0.09381

Table 1: The absolute neutrino masses in eV, estimated from oscillation data, using calculated $\psi = 0.031667$ as defined in the text.

neutrino oscillation observational data [8] on solar and atmospheric neutrino mass-squared differences. $\Delta m_{12}^2 = (m_2^2 - m_1^2) = 7.60 \times 10^{-5} eV^2$ and $|\Delta m_{32}^2| = |m_3^2 - m_2^2| = 2.40 \times 10^{-3} eV^2$, and define the following parameters $\rho = \frac{|\Delta m_{23}^2|}{m_3^2}$ and $\psi = \frac{\Delta m_{21}^2}{|\Delta m_{23}^2|}$, where m_3 is the input quantity allowed by the latest cosmological bound. For QDN in normal hierarchy (QD-NH) pattern, the other two mass eigenvalues are estimated from,

$$m_2 = m_3 \sqrt{1 - \rho}; m_1 = m_3 \sqrt{1 - \rho(1 + \psi)} \quad (6)$$

and for QDN in inverted hierarchy (IH-QD) the mass eigenvalues are extracted from,

$$m_2 = m_3 \sqrt{1 + \rho}; m_1 = m_3 \sqrt{1 + \rho(1 - \psi)}. \quad (7)$$

For suitable input value of m_3 , one can estimate the numerical values of m_1 and m_2 for both QD-NH and QD-IH cases, using the observational values of $|\Delta m_{23}^2|$ and Δm_{21}^2 . Table-1 gives the calculated numerical values for both models [9].

In the next step we parameterize the mass matrix in eq.(1) into three different types [5,10]:

Type IA with $D = \text{Diag}(m_1, -m_2, m_3)$. The mass matrix of this type can be parameterized using two parameters (ϵ, η) :

$$m_{LL} = \begin{pmatrix} \epsilon - 2\eta & -c\epsilon & -c\epsilon \\ -c\epsilon & \frac{1}{2} - d\eta & -\frac{1}{2} - \eta \\ -c\epsilon & -\frac{1}{2} - \eta & \frac{1}{2} - d\eta \end{pmatrix} m_3. \quad (8)$$

This predicts the solar mixing angle,

$$\tan 2\theta_{12} = -\frac{2c\sqrt{2}}{1 + (d-1)\frac{2}{\epsilon}}. \quad (9)$$

Different parameters	QD-NH		QD-IH	
	Type-IA	Type-IB	Type-IA	Type-IB
c	1.0	1.0	1.0	1.0
d	1.0	1.0	1.0	1.0
m_3	0.10	0.10	0.08	0.08
ϵ	0.57972	0.0015	0.78004	0.00169
η	0.14602	0.0649	0.19628	-0.08546
m_1 (eV)	0.08674	0.08675	0.09340	0.09340
m_2 (eV)	-0.08717	0.08717	-0.09380	0.09380
m_3 (eV)	0.10	0.10	0.08	0.08
$\sum m_i $ eV	0.27	0.274	0.267	0.274
Δm_{21}^2 eV ²	7.6×10^{-5}	7.6×10^{-5}	7.6×10^{-5}	7.6×10^{-5}
$ \Delta m_{23}^2 $ eV ²	2.2×10^{-3}	2.4×10^{-3}	2.4×10^{-3}	2.4×10^{-3}
$\tan^2 \theta_{12}$	0.50	0.50	0.50	0.50
$ m_{ee} $ eV	0.08688	0.0869	0.09354	0.09354

Table 2: Predictions for $\tan^2 \theta_{12} = 0.50$ and other parameters consistent with observations.

When we choose the constant parameters $c = d = 1.0$, we set for the tri-bimaximal mixings (TBM) $\tan 2\theta_{12} = -2\sqrt{2}$ (which is $\tan^2 \theta_{12} = 0.50$) and the values of ϵ and η are calculated for both QD-NH and QD-IH models, by using the values of observational data [8] in these two eigenvalue expressions: $m_1 = (2\epsilon - 2\eta)m_3$ and $m_2 = (-\epsilon - 2\eta)m_3$ obtained for TBM solution in eq.(8). The results are given in Table-2 for $\tan^2 \theta_{12} = 0.50$. The solar mixing angle can be further lowered by taking the values $c < 1$ and $d > 1$, while retaining the earlier values of ϵ and η extracted for TBM solution. For $\tan^2 \theta_{12} = 0.45$, case the results are shown in Table-3.

Type-IB with $D = \text{Diag}(m_1, m_2, m_3)$: This class of quasi-degenerate mass pattern is given by the mass matrix,

$$m_{LL} = \begin{pmatrix} 1 - \epsilon - 2\eta & c\epsilon & c\epsilon \\ c\epsilon & 1 - d\eta & -\eta \\ c\epsilon & -\eta & 1 - d\eta \end{pmatrix} m_3. \quad (10)$$

This predicts the solar mixing angle,

$$\tan 2\theta_{12} = \frac{2c\sqrt{2}}{1 + (1-d)\frac{2}{\epsilon}}. \quad (11)$$

Different parameters	QD-NH		QD-IH	
	Type-IA	Type-IB	Type-IA	Type-IB
c	0.868	0.945	0.868	0.96
d	1.025	0.998	1.0	1.002
m_3	0.10	0.10	0.08	0.08
ϵ	0.6616	0.00145	0.88762	0.00169
η	0.1655	0.06483	0.22317	-0.08546
m_1 (eV)	0.0876	0.08676	0.09392	0.09341
m_2 (eV)	-0.0880	0.08717	-0.09432	0.09381
m_3 (eV)	0.0996	0.10002	0.08	0.080014
$\sum m_i $ eV	0.274	0.274	0.268	0.267
Δm_{21}^2 eV ²	7.7×10^{-5}	7.3×10^{-5}	7.6×10^{-5}	7.4×10^{-5}
$ \Delta m_{23}^2 $ eV ²	2.2×10^{-3}	2.4×10^{-3}	2.4×10^{-3}	2.4×10^{-3}
$\tan^2 \theta_{12}$	0.45	0.45	0.45	0.45
$ m_{ee} $ eV	0.0877	0.08688	0.09403	0.09354

Table 3: Predictions for $\tan^2 \theta_{12} = 0.45$ and other parameters consistent with observations.

which gives the TBM solar mixing angle with the input values $c = 1$ and $d = 1$. When $\epsilon = 0$, $\eta = 0$, this leads to $m_{LL}^{diag} = diag(1, 1, 1)m_3$. Like in Type-IA, here ϵ and η values are computed for QD-NH and QD-IH, by using observational data [8] in the corresponding two eigenvalue expressions: $m_1 = (1 - 2\epsilon - 2\eta)m_3$ and $m_2 = (1 + \epsilon - 2\eta)m_3$ for TBM solution in eq.(10).

Type-IC with $D = \text{Diag}(m_1, m_2, -m_3)$: It is not necessary to treat this model separately as it is similar to Type-IB except with the interchange of two matrix elements (m_{22}) and (m_{23}) in the mass matrix in eq.(10), and this effectively imparts an additional odd CP-parity on the third mass eigenvalue m_3 in Type-IC. Such change does not alter the predictions of Type-IB. Tables 2 and 3 present our numerical results for $\tan^2 \theta_{12} = 0.50$ and $\tan^2 \theta_{12} = 0.45$ respectively which are consistent with cosmological bound.

3 Predictions of baryon asymmetry

We now apply the above Quasi-degenerate neutrino mass matrices with the values of the parameters already fixed in section 2, in the calculation of baryon asymmetry of the universe. This gives the second stage for the dis-

Type	(m,n)	M_1 (GeV)	M_2 (GeV)	M_3 (GeV)
NH-IA	(6,2)	4.8659×10^8	-3.5068×10^{12}	9.1256×10^{14}
	(8,4)	3.9414×10^6	-3.9774×10^{10}	6.0097×10^{13}
NH-IB	(6,2)	1.8117×10^8	2.6219×10^{12}	3.2528×10^{14}
	(8,4)	1.4994×10^6	2.1238×10^{10}	3.2527×10^{14}
IH-IA	(6,2)	4.5568×10^8	-2.8771×10^{12}	1.3273×10^{14}
	(8,4)	3.6910×10^6	-2.3241×10^{10}	1.8289×10^{14}
IH-IB	(6,2)	1.7153×10^8	2.8234×10^{12}	3.5098×10^{14}
	(8,4)	41.393×10^6	2.4809×10^{10}	3.8091×10^{14}

Table 4: Heavy right-handed Majorana neutrino mass M_j for QDN with normal and inverted ordering mode for $\tan^2 \theta_{12} = 0.45$, using neutrino mass matrices given in section 2. The entry (m,n) indicates the type of Dirac neutrino mass matrix, as explained in the text.

crimination among the QDN models.

For the numerical calculation of baryon asymmetry, we refer to all the relevant expressions given in the Appendix. First we translate mass matrices m_{LL} via the inversion of the seesaw formula [11], $M_{RR} = -m_{LR}^T m_{LL}^{-1} m_{LR}$. We choose a basis for U_R where $M_{RR}^{diag} = U_R^T M_{RR} U_R = diag(M_1, M_2, M_3)$, with real and positive mass eigenvalues. We then transform diagonal form of Dirac neutrino mass matrix, $m_{LR} = diag(\lambda^m, \lambda^n, 1)\nu$ to the U_R basis: $m_{LR} \rightarrow m'_{LR} = m_{LR} U_R Q$ where $Q = diag(1, e^{i\alpha}, e^{i\beta})$ is the complex matrix containing CP-violating Majorana phases introduced by hand. Here λ is the Wolfenstein parameter and the choice (m,n) in m_{LR} gives the type of Dirac neutrino mass matrix. The value of the vev is taken as $v = 174$ GeV.

At the moment we consider phenomenologically two possible forms of Dirac neutrino mass matrices such as (i) (m,n) = (6,2) for the charged-lepton type mass matrix, and (ii) (8,4) for up-quark type mass matrix. In this prime basis the Dirac neutrino Yukawa coupling becomes $h = \frac{h'_{LR}}{\nu}$ which enters in the expression of CP-asymmetry ϵ in (14) and (23) given in the Appendix. The new Yukawa coupling matrix h also becomes a complex quantity, and hence the term $Im(h^\dagger h)_{1j}$ appearing in lepton asymmetry ϵ_1 , gives a non-zero contribution. A straight forward simplification shows that $(h^\dagger h)_{1j}^2 = (Q_{22}^*)^2 Q_{22}^2 R_2 + (Q_{11}^*)^2 Q_{33}^2 R_2$ where $R_{2,3}$ are real parameters. After inserting the values of phases, the above expression leads to $Im(h^\dagger h)_{1j}^2 = -[R_2 \sin 2(\alpha - \beta) + R_3 \sin 2\alpha]$ which imparts a non-zero CP asymmetry for

<i>Type</i>	(m,n)	$(h^\dagger h)_{11}$	ϵ_1	η_{1B}	η_{3B}
NH-IA	(6,2)	3.73×10^{-6}	1.92×10^{-7}	9.07×10^{-12}	2.11×10^{-11}
	(8,4)	3.03×10^{-8}	1.55×10^{-9}	7.32×10^{-14}	1.71×10^{-13}
NH-IB	(6,2)	5.31×10^{-7}	1.12×10^{-14}	1.41×10^{-18}	5.67×10^{-13}
	(8,4)	4.30×10^{-9}	8.87×10^{-17}	1.12×10^{-20}	4.71×10^{-15}
IH-IA	(6,2)	3.77×10^{-6}	1.94×10^{-7}	8.50×10^{-12}	1.98×10^{-11}
	(8,4)	3.05×10^{-8}	1.57×10^{-9}	6.88×10^{-14}	1.60×10^{-13}
IH-IB	(6,2)	5.31×10^{-7}	9.75×10^{-15}	1.15×10^{-18}	5.95×10^{-13}
	(8,4)	4.30×10^{-9}	7.80×10^{-17}	9.17×10^{-21}	4.76×10^{-15}

Table 5: Values of CP asymmetry ϵ_1 and baryon asymmetry (η_{1B}, η_{3B}) for all quasi-degenerate models, with $\tan^2 \theta_{12} = 0.45$, using neutrino mass matrices given in the text. The entry (m,n) in m_{LR} indicates the type of Dirac neutrino mass matrix taken as charged lepton mass matrices (6,2) or up quark mass matrix (8,4) as explain in the text.

particular choice of (α, β) .

In our numerical estimation of lepton asymmetry, we chose some arbitrary values of α and β other than $\frac{\pi}{2}$ and 0. For example, light neutrino masses $(m_1, -m_2, m_3)$ of Type-IA model, leads to $M_{RR}^{diag} = \text{diag}(M_1, -M_2, M_3)$, and we thus fix the Majorana phase $Q = \text{diag}(1, e^{i\alpha}, e^{i\beta}) = \text{diag}(1, e^{i(\pi/2+\pi/4)}, e^{i\pi/4})$ for $\alpha = (\pi/2 + \pi/4)$ and $\beta = \pi/4$. The extra phase $\pi/2$ in α absorb the negative sign before heavy Majorana mass. In our search programmes such a choice of the phase leads to highest numerical estimation of lepton CP asymmetry.

In Table 4 we give numerical prediction on three heavy right-handed Majorana neutrino masses from these neutrino mass models under consideration for the case of $\tan^2 \theta_{12} = 0.45$. The three heavy right-handed Majorana mass matrices which are constructed through the inversion of Type-I seesaw mechanism, for two choices of diagonal Dirac-neutrino mass matrix discussed before. The corresponding baryon asymmetries η_B are estimated for both non-flavour η_{1B} and flavour η_{3B} leptogenesis respectively in Table 5. As expected, there is an enhancement in baryon asymmetry by a magnitude of order one (approximately) when flavour dynamics is included (see Table 5 in Type-IA model with charged lepton mass matrix) [12,13]. Type-IA with charged lepton Dirac neutrino mass matrix, is the only model sensitive and

$Type$	(m,n)	$T_R^{min} < T^R \leq T_R^{max}(GeV)$	$M_\phi^{min} < M_\phi \leq M_\phi^{max}(GeV)$
NH-IA	(6,2)	$5.51 \times 10^6 < T_R \leq 4.87 \times 10^7$	$9.73 \times 10^9 < M_\phi \leq 8.59 \times 10^{10}$
	(8,4)	$2.21 \times 10^2 < T_R \leq 3.94 \times 10^4$	$7.88 \times 10^6 < M_\phi \leq 1.40 \times 10^9$
NH-IB	(6,2)	$3.7 \times 10^8 < T_R \leq 1.85 \times 10^6$	$3.70 \times 10^8 < M_\phi \leq 1.89 \times 10^2$
	(8,4)	$2.96 \times 10^{13} < T_R \leq 1.49 \times 10^4$	$2.99 \times 10^6 < M_\phi \leq 1.52 \times 10^{-2}$
IH-IA	(6,2)	$5.12 \times 10^5 < T_R \leq 4.56 \times 10^6$	$9.11 \times 10^8 < M_\phi \leq 8.11 \times 10^9$
	(8,4)	$5.12 \times 10^5 < T_R \leq 3.69 \times 10^4$	$7.38 \times 10^6 < M_\phi \leq 5.32 \times 10^5$
IH-IB	(6,2)	$3.84 \times 10^{12} < T_R \leq 1.72 \times 10^7$	$3.44 \times 10^8 < M_\phi \leq 1.54 \times 10^2$
	(8,4)	$3.88 \times 10^{12} < T_R \leq 1.39 \times 10^4$	$2.79 \times 10^6 < M_\phi \leq 9.99 \times 10^{-3}$

Table 6: Theoretical bound on reheating temperature T_R and inflaton masses M_ϕ in non-thermal leptogenesis, calculated using data from Table 5, for all neutrino mass models with $\tan^2 \theta_{12} = 0.45$.

consistent with data on observed baryon asymmetry.

In case of non-thermal leptogenesis, the lightest right-handed Majorana neutrino mass M_1 from Table 4 and the CP asymmetry ϵ_1 from Table 5, for all the neutrino mass models, are used in the calculation of the bounds: $T_R^{min} < T_R \leq T_R^{max}$ and $M_\phi^{min} < M_\phi \leq M_\phi^{max}$ in Table 6. The baryon asymmetry $Y_B = \frac{n_B}{s} = 8.7 \times 10^{-11}$ is taken as input value from WMAP observational data. Only those neutrino mass models which simultaneously satisfy the two constraints, $T_R^{max} > T_R^{min}$ and $M_\phi^{max} > M_\phi^{min}$, could survive in the non-thermal leptogenesis scenario. The predicted inflaton mass as $M_\phi \sim 10^{11}$ GeV for reheating temperature $T_R = 10^6$ GeV are needed to produce the observed baryon asymmetry of the universe [14,15]. From Table 6, the neutrino mass models with (m,n) which are compatible with $M_\phi \sim (10^{10} - 10^{13})$ GeV and $T_R \approx (10^6 - 10^7)$ GeV are listed as NH-IA (6,2) and IH-IA (6,2) only. Again in order to avoid gravitino problem [8] in supersymmetric models, one has the bound on reheating temperature, $T_R \approx (10^6 - 10^7)$ GeV. This implies that Type-IA where Dirac neutrino mass matrix is taken as charged lepton mass matrix is the only model consistent with the observed baryon asymmetry. These findings nearly agree with flavoured thermal leptogenesis for Type NH-IA (6,2) model in Table 5. This result is also consistent with quasi-degenerate in inverted hierarchical (IA) models for charged lepton mass matrix (6,2) [15].

4 Summary and Conclusions

We have studied the effects of Majorana CP phases on the prediction of absolute neutrino masses in three types of QDN models having both normal and inverted hierarchical patterns within μ - τ symmetry. These predictions are consistent with data on the mass squared difference derived from various oscillation experiments, and from the upper bound on absolute neutrino masses in $0\nu\beta\beta$ decay as well as cosmological upper bound of $\sum_i m_i \leq 0.28$ eV. It has been found that QDN models with $m_i \leq 0.09$ eV, are still far from discrimination and hence the quasi-degenerate pattern is not yet ruled out. The prediction on solar mixing angle is also found to be lower than TBM value viz, $\tan^2 \theta_{12} = 0.45$ which coincides with the best-fit in the neutrino oscillation data [8].

In the next stage, left-handed Majorana neutrino mass matrices m_{LL} have been employed to estimate the baryon asymmetry of the universe, in both thermal and non-thermal leptogenesis scenario (Tables 5-6). We use the CP violating Majorana phases derived from right-handed Majorana mass matrix, and also Dirac neutrino mass matrix as either charged lepton mass matrix or up-quark mass matrix. We also observe some enhancement effects in flavour leptogenesis [20] compared to non-flavour leptogenesis by a magnitude of order one. The predicted inflaton mass needed to produce the observed baryon asymmetry of the universe is found to be $M_\phi \sim 10^{10}$ GeV corresponding to the reheating temperature $T_R = 10^6$ GeV [14,15]. The analysis shows that quasi-degenerate model in normal hierarchical pattern (NH-IA) with charged lepton Dirac mass matrix (6,2), appears to be a favourable choice in nature. The quasi-degenerate model in inverted hierarchy (IH-IA) with the charged lepton Dirac mass matrix (6,2), is not completely ruled out. The results presented in this article are new and have subtle hints in the discrimination of neutrino mass models. This could establish the quasi-degenerate neutrinos as natural physical neutrinos in the neutrino oscillation experiments.

Acknowledgement

Ng.K.F thanks the University Grants Commission, Government of India for sanctioning the project entitled “**Neutrino masses and mixing angles in neutrino oscillations**” vide Grant. No 32-64/2006 (SR). This work was carried out through this project.

Appendix

Thermal leptogenesis

The light left-handed Majorana neutrino mass matrix m_{LL} and heavy right-handed Majorana mass matrix M_{RR} are related through the canonical seesaw formula (known as Type I) [11] in a simple way:

$$m_{LL} = -m_{LR}M_{RR}m_{LR}^T \quad (12)$$

where m_{LR} is the Dirac neutrino mass matrix. For our calculation of lepton asymmetry, we consider the model [16,17] where the asymmetry decay of the lightest of the heavy right-handed Majorana neutrinos, is assumed. The physical Majorana neutrino N_R decays into two modes: $N_R \rightarrow l_L + \bar{\varphi}$, $N_R \rightarrow \bar{l}_L + \varphi$ where l_L is the lepton and \bar{l}_L is the antilepton; φ and $\bar{\varphi}$ are the Higgs and anti-Higgs particles respectively. The branching ratio for these two decay modes are likely to be different. The CP asymmetry which is caused by the interference of tree level with one-loop corrections for the decays of the lightest of heavy right-handed Majorana neutrino N_R is defined by [16,18]

$$\epsilon = \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}} \quad (13)$$

Here $\Gamma = \Gamma(N_i \rightarrow l_L \bar{\varphi})$ and $\bar{\Gamma} = \Gamma(N_i \rightarrow \bar{l}_L \varphi)$ are the decay rates. A perturbative calculation from the interference between tree level and vertex plus self energy diagrams, gives [19] the lepton asymmetry ϵ_1 for non-SUSY case as

$$\epsilon_i = \frac{1}{8\pi} \frac{1}{(h^\dagger h)_{ii}} \sum_{j=2,3} \text{Im}[(h^\dagger h)_y]^2 [f(\frac{M_j^2}{M_i^2})g(\frac{M_j^2}{M_i^2})] \quad (14)$$

where $f(x)$ and $g(x)$ represent the contributions from vertex and self energy corrections respectively, $f(x) = \sqrt{x}[-1 + (x+1)\ln(1+\frac{1}{x})]$, $g(x) = \frac{\sqrt{x}}{x-1}$. For hierarchical right-handed neutrino masses where x is large, we have the approximation [16], $f(x) + g(x) \cong \frac{3}{2\sqrt{x}}$. This simplifies to

$$\epsilon_i = -\frac{3}{16\pi} [\frac{\text{Im}[(h^\dagger h)_{12}^2]}{(h^\dagger h)_{11}} \frac{M_1}{M_2} + \frac{\text{Im}[(h^\dagger h)_{13}^2]}{(h^\dagger h)_{11}} \frac{M_1}{M_3}] \quad (15)$$

where $h = \frac{m_{LR}}{\nu}$ is the Yukawa coupling of the Dirac neutrino mass matrix in the diagonal basis of M_{RR} and $\nu = 174$ GeV is the vev of the standard model.

In term of light Majorana neutrino mass matrix m_{LL} the above expression can be simplified to

$$\epsilon_1 = -\frac{3}{16\pi} \frac{M_1}{(h^\dagger h)_{11}} \text{Im}[(h^\dagger m_{LL} h^*)_{11}] \quad (16)$$

For quasi-degenerate spectrum of the heavy right-handed Majorana neutrino masses, i.e., for $M_1 \approx M_2 < M_3$ the asymmetry is largely enhanced by a resonance factor and in such situation, the lepton asymmetry is modified [21] to

$$\epsilon_1 = \frac{1}{8\pi} \frac{\text{Im}[(h^\dagger h)_{12}^2]}{(h^\dagger h)_{11}} R \quad (17)$$

where $R = \frac{M_2^2(M_2^2 - M_1^2)}{(M_1^2 - M_2^2)^2 + \Gamma_2^2 M_1^2}$ and $\Gamma_2 = \frac{(h^\dagger h)_{22} M_2}{8\pi}$. It can be noted that in case of SUSY, the functions $f(x)$ and $g(x)$ are given by $f(x) = \sqrt{x} \ln(1 + \frac{1}{x})$ and $g(x) = \frac{2\sqrt{x}}{x-1}$; for large x one can have $f(x) + g(x) \approx 3/\sqrt{x}$. Therefore the factor $3/8$ will appear in place of $3/16$ in the expression of CP asymmetry [20] in eq. (15). The CP asymmetry parameter ϵ_i is related to leptonic asymmetric parameter Y_L as

$$Y_L \approx \frac{n_L - \bar{n}_L}{s} = \sum_i^3 \frac{\epsilon_i \kappa_i}{g_{*i}} \quad (18)$$

where n_L is the lepton number density, \bar{n}_L is the anti-lepton number density, s is the entropy density, κ_i is the dilution factor for the CP asymmetry ϵ_i , and g_{*i} is the effective number of degrees of freedom at temperature $T = M_i$. The baryon asymmetry Y_B produced through the sphaleron transition of Y_L while the quantum number B-L remains conserved, is given by [22]

$$Y_B = \frac{n_B}{s} = C Y_{B-L} = C Y_L \quad (19)$$

where $C = \frac{8N_F + 4N_H}{22N_F + 13N_H}$, N_F is the number of fermion families and N_H is the number of Higgs doublets. Since $s = 7.04 n_\gamma$ the baryon number density over photon number density n_γ corresponds to the observed baryon asymmetry of the universe [23],

$$\eta_B^{SM} = \left(\frac{n_B}{n_\gamma}\right)^{SM} \approx d \kappa_1 \epsilon_1 \quad (20)$$

where $d \approx 0.98 \times 10^{-2}$ is used in the present calculation. In case of MSSM, there is no major numerical change with respect to the non-supersymmetric

case in the estimation of baryon asymmetry. One expects approximate enhancement factor of about $\sqrt{2}\sqrt{2}$ for strong (weak) washout regime [20].

In the expression for baryon-to-photon ratio in eq. (20), κ_1 describes the washout factor of the lepton asymmetry due to various lepton number violating processes. This efficiency factor (also known as dilution factor) mainly depends on the effective neutrino mass

$$\widetilde{m}_1 = \frac{(h^*h)_{11}\nu^2}{M_1} \quad (21)$$

where ν is the electroweak vev, $\nu = 174$ GeV. For $10^{-2}eV < \widetilde{m}_1 < 10^3eV$, the washout factor κ_1 can be well approximately by [19,24]

$$\kappa_1(\widetilde{m}_1) = 0.3 \left[\frac{10^{-3}}{\widetilde{m}_1} \right] [\log \frac{\widetilde{m}_1}{10^{-3}}]^{-0.6} \quad (22)$$

The value of κ_1 is valid only for the given range of \widetilde{m} [25].

Flavoured thermal leptogenesis

It is inevitable to include the flavour effects in thermal leptogenesis [26] and study its effects on the enhancement in baryon asymmetry over the single flavour approximation. In the flavour basis the equation for lepton asymmetry in $N_1 \rightarrow l_\alpha \varphi$ decay where $\alpha = (e, \mu, \tau)$ becomes

$$\epsilon_{\alpha\alpha} = \frac{1}{8\pi} \frac{1}{(h^\dagger h)_{11}} \left[\sum_{j=2,3} \text{Im}[h_{\alpha 1}^* (h^\dagger h)_{1j} h_{\alpha j}] g(x_j) + \sum_j \text{Im}[h_{\alpha 1}^* (h^\dagger h)_{j1} h_{\alpha j}] \frac{1}{(1-x_j)} \right] \quad (23)$$

Here we have $x_j = \frac{M_j^2}{M_1^2}$ and $g(x_j) \sim \frac{3}{2} \frac{1}{\sqrt{x_j}}$. The efficiency factor for the out-of-equilibrium situation is given by $\kappa_\alpha = \frac{m_*}{m_{\alpha\alpha}}$. Here $\frac{8\pi H \nu^2}{M_1^2} \sim 1.1 \times 10^{-3}$ eV, and $m_{\alpha\alpha} = \frac{h_{\alpha 1}^\dagger h_{\alpha 1}}{M_1} \nu^2$. This leads to the baryon asymmetry of the universe,

$$\eta_{3B} = \frac{\eta_B}{\eta_\gamma} \sim 10^{-2} \sum_\alpha \epsilon_{\alpha\alpha} \kappa_\alpha \sim 10^{-2} m_* \sum_\alpha \frac{\epsilon_{\alpha\alpha}}{\widetilde{m}_{\alpha\alpha}} \quad (24)$$

For single flavour case, the second term in $\epsilon_{\alpha\alpha}$ vanishes when summed over all flavours. Thus

$$\epsilon_1 \equiv \sum_\alpha \epsilon_{\alpha\alpha} = \frac{1}{8\pi} \frac{1}{(h^\dagger h)_{11}} \sum_j \text{Im}[(h^\dagger h)_{1j}^2] g(x_j). \quad (25)$$

This leads to baryon asymmetry,

$$\eta_{1B} \approx 10^{-2} m_* \frac{\eta_1}{\widetilde{m}} = 10^{-2} \kappa_1 \epsilon_1 \quad (26)$$

where $\epsilon_1 = \sum_{\alpha} \epsilon_{\alpha\alpha}$ and $\widetilde{m} = \sum_{\alpha} \widetilde{m}_{\alpha\alpha}$.

Non-thermal leptogenesis

We now focus our attention on the non-thermal leptogenesis scenario [27] where the right-handed neutrinos are produced through the direct non-thermal decay of the inflaton ϕ . We follow the standard procedure [28] where non-thermal leptogenesis and baryon asymmetry in the universe had been studied in different neutrino mass models whereby some mass models were excluded using bound from below and from above on inflaton mass and reheating temperature after inflation. We start with the inflaton decay rate given by

$$\Gamma_{\phi} = \Gamma(\phi \rightarrow N_i N_i) \approx \frac{|\lambda_i|^2}{4\pi M_{\phi}} \quad (27)$$

where λ_i are the Yukawa coupling constants for the interaction of three heavy right-handed neutrinos N_i with the inflaton ϕ of mass M_{ϕ} . The reheating temperature after inflation is given by the expression,

$$T_R = \left(\frac{45}{2\pi^2 g_*}\right)^{\frac{1}{4}} (\Gamma_{\phi} M_p)^{\frac{1}{2}} \quad (28)$$

where $M_p \approx 2.4 \times 10^{18}$ GeV is the reduced Planck mass [29] and g_* is the effective number of relativistic degrees of freedom at reheating temperature. For SM we have $g_* = 106.75$, and for MSSM $g_* = 228.75$. If the inflaton dominantly couples to N_i , the branching ratio of this decay process is taken as $\text{BR} \sim 1$, and the produced baryon asymmetry of the universe can be calculated by the following relation [30],

$$Y_B = \frac{n_B}{s} = C Y_L = C \frac{3}{2} \frac{T_R}{M_{\phi}} \epsilon_1 \quad (29)$$

where Y_L is the lepton asymmetry generated by CP-violating out-of-equilibrium decays of heavy neutrino N_1 and T_R is the reheating temperature. The fraction C has the value $C = -28/79$ for SM and $C = -8/15$ in the

MSSM. The observed baryon asymmetry measured in WMAP data, $\eta_B = \frac{\eta_B}{\eta_\gamma} = 6.5 \times 10^{-10}$ [3], where $s=7.04 n_\gamma$ is related to $Y_B = n_B/s = 8.6 \times 10^{-11}$ in eq.(29). From eq.(29) the connection between T_B and M_ϕ is expressed as,

$$T_R = \left(\frac{2Y_B}{3C\epsilon_1}\right)M_\phi \quad (30)$$

The above expression is supplemented by two more boundary conditions [28]: (i) lower bound on inflaton mass, $M_\phi > 2M_1$ coming from allowed kinematics of inflaton decay to two right-handed Majorana neutrinos N_1 , and (ii) an upper bound for the reheating temperature, $T_R \leq 0.01M_1$ coming from out-of-thermal equilibrium decay of N_1 . Using the observed central value of the baryon asymmetry Y_B and theoretical prediction of CP asymmetry ϵ_1 in eq.(30), one can establish the relation between T_R and M_ϕ for each neutrino mass model. The lightest right-handed neutrino mass M_1 and the CP asymmetry ϵ_1 neutrino mass models are used in the calculation of theoretical bounds: $T_R^{min} < T_R \leq T_R^{max}$ and $M_\phi^{min} < M_\phi \leq M_\phi^{max}$ following eq.(19) along with other two boundary conditions cited above. Only those models which satisfy the constraints $T_R^{max} > T_R^{min}$ and $M_\phi^{min} < M_\phi^{max}$ simultaneously can survive in the non-thermal leptogenesis.

References

- [1] Shaun A. Thomas, Filipe B. Abdalla, Ofer Lahav, "Upper Bound on 0.28 eV on Neutrino Masses from the Largest Photometric Redshift Survey," Phys Rev. Lett, Vol.**105**, 2010, pp. 031301; Jubien Lesgoues, "Galaxies Weigh in on neutrinos," Physics, Vol **3**, 2010, pp. 57-59.
- [2] S. Pascoli, S.T. Petcov, "Majorana Neutrinos, Neutrino Mass Spectrum and the $\langle M \rangle \sim 0.001$ eV Frontier in Neutrinoless Double Beta Decay," Phys. Rev. D, Vol.**77** 2008, pp.113003, and further references there in.
- [3] D.N Spergel et.al., Astrophysics. J. Suppl. **148**, 175 (2003).
- [4] P.F. Harrison, W.G. Scott, 2002, " $\mu - \tau$ Reflection Symmetry in Lepton Mixing and Neutrino Oscillation," Phys. Lett. B, **547**, pp. 219-228; Lam, C.S., 2005, "Neutrino $\mu - \tau$ symmetry and Inverted Hierarchy," Phys. Rev. D., **71**, pp. 093001; Grimus, W., Lavoura, L., 2007, "A Three-Parameters Models for the Neutrino Mass Matrix," J. Phys. G **34**(7), pp. 1757-1769.

- [5] G. Altarelli, F. Feruglio, “Neutrino Masses and Mixings: A Theoretical Perspective,” *Phys. Rep.*, Vol. **320**, 1999, pp. 295-325.
- [6] Z. Maki, M. Nakagawa, S. Sakata, “Remarks on the Unified Model of Elementary Particles, ” *Prog. Theor. Phys.*, Vol. **28**, 1962, pp. 870-878; B. Pontecorvo, “Neutrino Experiments and the Problems of Conservation of lepton Charge,” *Zh. Eksp. Teor. Fiz.* Vol.**53**, 1968, pp. 1717-1725 [*Sov. Phys. JETP.* Vol. **26**, 1968, pp. 984-988].
- [7] P.F. Harrison, D.H. Perkins, W.G. Scott, “Tri-Bimaximal Mixing and the Neutrino Oscillation Data,” *Phys. Lett. B*, Vol. **530**, 2002, pp. 167-178. P.F. Harrison, W.G. Scott, “Permutation Symmetry, Tri-Bimaximal Neutrino Mixing and S_3 ,” *Phys. Lett. B*, Vol. **557**, 2003, pp. 76-90.
- [8] B.T. Cleveland et al., *Astrophysics. J.* **496** (1998) 505; J.N. Abdurashitov et al [SAGE Collaboration], *J. Exp. Theor. Phys* **95** (2002) 181 [astro-ph/0204245]; T. Kirsten et al [GALLEX and GNO Collaborations], *Nucl Phys. B Suppl.* **118** (2003) 33; C. Cattadori, N. Ferri and L. Pandola, *Nucl. Phys. B (Proc. Suppl.)* **143** (2005) 3; T. Schetz, M. Tortola, J.W.F. Valle, “Three-Flavour Neutrino Oscillation Update,” *New J. Phys.* Vol. **10**, 2008, pp. 113011.; “Global Neutrino Data and Recent Reactor Fluxes: Status of Three-Flavour Oscillation Parameters,” *New J. Phys.* Vol. **13**, 2011, pp. 063004. M.C. Gonzales-Garcia, M. Maltoni, “Phenomenology with Massive neutrinos,” *Phys Rep.* Vol. **460**. 2008; J.W.F. Valle. “Understanding and Probing Neutrino Oscillation.” Invited Talk in Neutrino-2010, Athens, June 14, 2010; A. Bandyopadhyaya et al., “Physics at a Future Neutrino Factory and Super-Beam Facility,” *Rep. Prog. Phys.* Vol. **72**, 2009, pp. 106201; S.T. Petcov, presented at 2011BCVSPIN, Hue, Vietnam, 27 July 2011, “Massive Neutrinos, Neutrinos Mixing, Oscillation, Lepton CP-Violation and Beyond.”
- [9] Ngouniba. Ki Francis and Ngangkham. Nimai Singh, “Quasi-Degenerate Neutrino Masses with Normal and Inverted Hierarchy,” *Journal of Modern Physics*, **Vol-2**, Number-11, November 2011, pp 1280-1284.
- [10] N.Nimai Singh, Monisa Rajkhowa, Abhijit Borah, “Lowering Solar Mixing Angle in Inverted Hierarchy without charged Lepton Corrections,” *J. Phys. G: Nucl. Part. Phys.* **Vol.34**, 2007, pp. 345-351.; “Deviation

From Tri- Bimaximal Mixing Through Flavour Twisters in Inverted and Normal Hierarchical Neutrino Mass Models,” *Pramana J. Phys*, **Vol.69**, 2007, pp. 533-549.

- [11] Gell-Mann, M., Ramond, P., and Slansky, R., 1979, “Supergravity, Proceeding of the Workshop,” Stony Brook, New York, 1979, Edited by Nieuwenhuizen, P. Van, and Freedman, D., (North-Holland, Amsterdam, 1979) Mahapatra, R.N., and Senjanovic, G., 1980, ”Neutrino Mass and Spontaneous Parity Non-conservation,” *Phys. Rev. Lett.*, **44(14)**, pp. 912-915.
- [12] Asmaa Abada, Sacha Davidson, Francois-Xavier Josse-Michaux, Marta Losada and Antonio Riotto, “Flavour Issues in Leptogenesis,” *arxiv: hep-ph/0601083v3*, 5 Jan 2007.
- [13] Enricho Nardi et al, “The importance of flavour in Leptogenesis,”
- [14] Takeshi Fuguyama, Tatsuru Kikuchi and Toshiyuki Osaka, “Non-thermal Leptogenesis and a prediction of Inflaton Mass in a Supersymmetric SO(10) Model,” **arxiv: hep-ph/053101v2 (2005)**.
- [15] Ng.K. Francis, N. Nimai Singh, H. Zeen Devi and Amal Kr Sarma, “Constraints on the Validity of Neutrino Mass Models through Thermal and Non-thermal leptogenesis,” *IJAP Vol-1* Number-2. (November 2011) pp. 145-160.
- [16] Fukugita, M., and Yanagita, T., 1986, “Baryogenesis without Grand Unification,” *Phys. Lett. B.*, **174(1)**, pp.45-47.
- [17] Luty, M.A. 1992, “Baryogenesis via Leptogenesis,” *Phys. Rev. D*, 1992, **45**, pp. 455-465.
- [18] Kolb, E.W., Turner, M.S., *The Early Universe*. Addison-Wesely, New York.
- [19] Covi, L., Roulet, E., and Vissani, F., 1996, “CP Violating Decay in Leptogenesis Scenarios,” *Phys. Lett. B.*, **384**, pp. 169-174; Pilaftis, A., 1997, “CP Violation and Baryogenesis due to Heavy Majorana Neutrinos,” *Phys. Rev. D.*, **56**, PP.54315451; Buchmuller, W., and Plumacher, M., 1998, ”CP Asymmetry in Baryogenesis Neutrino Decay,” *Phys. Lett. B.*, **431**, pp. 354-362.

- [20] Davidson. S., Nardi. E.N.Y., 2008, “Leptogenesis,” Phys. Report., **466**, pp. 105-177.
- [21] Pilaftis, A., 1997, “CP Violation and Baryogenesis due to Heavy Majorana Neutrinos,” Phys. Rev. D., **56**, pp. 5431-5451; Pilaftis, A., Underwood, T.E.J., 2004, “Resonant Leptogenesis,” Nucl. Phys. B., **692**, pp. 303-345.
- [22] Khlebnikov, S.Y., Shaposhnikov, M.E., 1998, “The Statistical Theory of Anomalous Fermion Number Non-conservation,” Nucl. Phys. B., **308**, pp. 885-912; Buchmuller, W., Pecci, R. D.m and Yanagida, T., 2007, “Leptogenesis as the Origin of Matter,” Ann. Rev. Nucl. Part. Sci. 55. pp 311-355.
- [23] Bari, P.D., 2005, “Seesaw Geometry and leptogenesis,” Nucl. Phys. B., **27** pp. 318-354; Buchmuller, W., Bari, P. D., and Plimacher, M., 2003, “The Neutrino Mass Window for Baryogenesis,” Nucl. Phys. B., **665**, pp. 445-468.
- [24] Branco, G. C., Felipe, R. G., Joaquim, F.R and Rebelo, M.N., 2002, “Leptogenesis, CP Violation and Neutrino Data: What can we learn,” Nucl. Phys. B., **640**:pp. 202-232; Akhmedov, E. K., Frigerio, M., and Smirnov, A.Y., 2003, JHEP, 0309, pp. 021-051; Adhikary, B., and Ghosal, A., 2008, “Neutrino $U(\epsilon_3)$, Cp Violation and leptogenesis in a seesaw Typw Softly Broken $A(4$ Symmetry Model,” Phys. Rev. D., **78**, pp. 073007; Buccella, F., Falcone, D., and Oliver, L., 2008, “Leptogenesis within a Generalized Quark-Lepton Symmetry,” Phys. Rev. D., **77**, pp. 033002.
- [25] Babu, K. S., Bachri, A., and Aissaoui, H, 2006., “Leptogenesis in minimal left-right symmetric models,” Nucl. Phys. B., **738**, pp. 76-92.
- [26] Adaba, A., Aissaoui, H., and Losada, M., 2005, “A Model of leptogenesis at the TeV scale,” Nucl.Phys. B., **728** pp. 55-66.; Vives, O., 2006. “FlavouredLeptogenesis: a successful Thermal Leptogenesis with N_1 Mass below 10^8 GeV,” Phys.Rev. D., **73** pp. 073006; Abadam A., Davidson, S., Ibarra, A., Josse-Michaux, F. X., Losada. M., Riotto, A., 2007, “Flavour matters in Leptogenesis,” JCAP, 0604, pp. 004-040;

- Nardi, E., Nir, Y., Roulet, E., and Racker, J., 2006, “The Important of Flavour in leptogenesis,” JHEP, 0601, pp. 164-171.
- [27] Lazarides, G., and Shafi, Q., 1991, “Origin of Matter in the Inflationary Cosmology,” Phys. Lett. B., **258** pp. 305-309; Kumekawa, K., Moroi, T., Yanagida, T., 1994, “Flat Potential for Inflaton with a Discrete R Invariance in Supergravity,” Prog. Theor. Phys., **92**, pp. 437-448; Giudice, G. F., Peloso, M., Riotto, A., 1999; “Production of Massive Fermions at Preheating and Leptogenesis,” JHEP., **9908**, pp. 014-043; Asaka, T., Hamaguchi, K., Kawasaki, M., and Yanagida, T., 1999, “Leptogenesis in Inflaton Decay,” Phys. Rev B., **464** pp. 12-18; “Leptogenesis in Inflationary Universe,” Phys. Rev. D., 2000, **61(8)** pp. 083512; Asaka, T., Nielsen, H. B., and Takanishi, Y., 2002, “Non-Thermal Leptogenesis from the Heavier Majorana Neutrinos,” Nucl. Phys. B., **647**, pp. 252-274; Mazumdar A., 2004, CMB Constraints on Non-Thermal Leptogenesis,” Phys. Lett. B., **580**, pp. 7-16; Fukuyama, T., Kikuchi, T., and Osaka, T., 2005, “Non-Thermal Leptogenesis and a Prediction of Inflaton Mass in a Supersymmetric SO(10) Model,” JCAP, 0506, pp. 005-014.
- [28] Nanotopolous, G., 2006, “Non-Thermal and Baryon Asymmetry in Different Neutrino Mass Models,” Phys. Lett. B., **643**, pp. 279-283.
- [29] Steffen, F.D., 2008, “ Probing the relating Temperature at Colliders and with Primordial Nucleosynthesis,” Phys. Lett. B., **669** pp. 74-80.
- [30] Buchmuller, W., Peccei, R.D., and Yanagida, T., 2005, “Leptogenesis as Origin of Matter,” Annual. Rev. Nucl. Part. Sci. **55**, pp. 311-355.